Systems of 3 Equations - Elimination Method

Elimination Method:

1. Pick a pair of equations. Eliminate one of the variables.
2. Pick another pair of equations. (Re-use one from step 1.) Eliminate the same variable as in step 1.
3. Solve the resulting system of two equations.
4. Find the final variable using one of the original equations.

If you obtain a false equation, such as 0 = 1, in any of the steps, then the system has no solution.

If you do not obtain a false equation, but obtain an identity such as 0 = 0, then the system has infinitely many solutions.

$3x + y - 2z = 10$
$2(6x - 2y + z = -2)$ ---->
$x + 4y + 3z = 7$

$3x + y - 2z = 10 \longrightarrow 3x + y - 2z = 10$
$12x - 4y + 2z = -4$
$15x - 3y = 6$

$3x + y - 2z = 10$
$-3(6x - 2y + z = -2)$ ---->
$x + 4y + 3z = 7$

$-18x + 6y - 3z = 6$
$x + 4y + 3z = 7$
$-17x + 10y = 13$

$10(15x - 3y = 6) \longrightarrow 150x - 30y = 60$
$3(-17x + 10y = 13) \longrightarrow -51x + 30y = 39$
$99x = 99$
$x = 1$

$-17(1) + 10y = 13$
$-17 + 10y = 13$
$10y = 30$
$y = 3$

$x + 4y + 3z = 7$
$1 + 4(3) + 3z = 7$
$13 + 3z = 7$
$3z = -1$
$z = -2$

$(1, 3, -2)$

Systems of 3 Equations - Substitution Method

1. Solve for one of the variables in one of the equations.
2. Substitute the expression for that variable into the other two equations.
3. Solve the resulting system of two equations.
4. Find the final variable using one of the original equations.

$x + y - z = 4$ $\xrightarrow{-y+z}$ $x = 4 - y + z$

$3x + 2y + 4z = 17$

$-x + 5y + z = 8$

$3(4-y+z) + 2y + 4z = 17$

$12 - 3y + 3z + 2y + 4z = 17$

$12 - 1y + 7z = 17$

$-1y + 7z = 5$

$-(4-y+z) + 5y + z = 8$

$-4 + y - z + 5y + z = 8$

$-4 + 6y = 8$

$6y = 12$

$y = 2$

$-1(2) + 7z = 5$

$7z = 7$

$z = 1$

$(3, 2, 1)$

$x = 4 - y + z$

$x = 4 - 2 + 1 = 3$